# 17216

**11718**

**3 Hours / 100 Marks** Seat No. | | | | | | | | |

*Instructions* – (1) All Questions are *Compulsory.*

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Use of Non-programmable Electronic Pocket Calculator is permissible.

(7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

**Marks**

**1. Attempt any TEN of the following:** **20**

a) Find $x$ and $y$ if $x(1-i)+y(2+i)+6=0$

b) Express in $a+ib$ form $\dfrac{2-\sqrt{3}\,i}{1+i}$

c) If $f(x)=x^2-2x+5$ and $t=y-2$, find $f(t)$.

d) If $f(x)=\log_a^x$, prove that $f(m)+f(n)=f(m.n)$

e) Evaluate $\lim\limits_{x\to -4} \dfrac{x^2+3x-4}{x^2+7x+12}$

f) Evaluate $\lim\limits_{x\to 0} \dfrac{4x-\tan x}{3x+\tan x}$

**Marks**

g) Evaluate $\lim_{x \to 0} \left(\frac{e^{3x} - 1}{4x}\right)$

h) Find $\frac{dy}{dx}$, if $y = \log[\tan(4 - 3x)]$

i) Find $\frac{dy}{dx}$, if $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$

j) Differentiate $\cos^{-1}(1 - 2\sin^2 x)$

k) Show that there exist a root of the equation $x^3 + 2x^2 - 8 = 0$ between 1 and 2.

l) Solve the following equations by using Gauss-Seidal method (only first iteration)

$10x + 2y + z = 9$; $x + 10y - z = -22$; $-2x + 3y + 10z = 22$

**2. Attempt any <u>FOUR</u> of the following:** **16**

a) Simplify using De-Moiver's theorem

$$\frac{(\cos\theta - i\sin\theta)^6 \ (\cos 5\theta - i\sin 5\theta)^{-2}}{(\cos 8\theta + i\sin 8\theta)\frac{1}{2}}$$

b) Find cube root of unity.

c) If $x + iy = \sin(A + iB)$, prove that $\frac{x^2}{\cos h^2 B} + \frac{y^2}{\sin h^2 B} = 1$

d) Prove that $(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n$

$= 2^{n+1} \cdot \cos^n\left(\frac{\theta}{2}\right) \cdot \cos\left(\frac{n\theta}{2}\right)$

e) If $f(x) = \frac{2x + 5}{3x - 4}$ and $t = \frac{5 + 4x}{3x - 2}$ show that $f(t) = x$

f) If $f(x) = \log\left(\frac{1 + x}{1 - x}\right)$, show that $f(a) + f(b) = f\left(\frac{a + b}{1 + ab}\right)$

**Marks**

**3. Attempt any FOUR of the following:** **16**

a) If $f(x) = \log\left(\frac{x}{x-1}\right)$, show that $f(a+1) + f(a) = \log\left(\frac{a+1}{a-1}\right)$

b) If $f(x) = x - \frac{1}{x}$, then prove that $[f(x)]^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$

c) Evaluate $\lim_{x \to 0} \left(\frac{\sqrt{1+x} - \sqrt{1-x}}{x}\right)$

d) Evaluate $\lim_{x \to \pi/4} \left(\frac{2 - \sec^2 x}{1 - \tan x}\right)$

e) Evaluate $\lim_{x \to 0} \left(\frac{6^x - 2^x - 3^x + 1}{x^2}\right)$

f) Evaluate $\lim_{x \to 5} \left(\frac{\log x - \log 5}{x - 5}\right)$

**4. Attempt any FOUR of the following:** **16**

a) Using first principal find the derivative of $\sin x$

b) Find $\frac{dy}{dx}$ if $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$

c) Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left(\frac{\cos x + \sin x}{\sqrt{2}}\right)$

d) If $e^y = y^x$, prove that $\frac{dy}{dx} = \frac{(\log y)^2}{(\log y - 1)}$

e) If $y = (\sin x)^{\log x}$, find $\frac{dy}{dx}$.

f) If $x^3 + y^3 = 3axy$, find $\frac{dy}{dx}$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$

**Marks**

**5. Attempt any FOUR of the following:** **16**

a) Evaluate $\lim_{x \to \infty} \left(\frac{1+3x}{3x-2}\right)^{2x}$

b) Evaluate $\lim_{x \to a} \left(\frac{\cos x - \cos a}{\sqrt{x} - \sqrt{a}}\right)$

c) Using Bisection method find the approximate root of $x^3 - x - 4 = 0$ (Three iterations only)

d) Using Regula-Falsi method, find approximate root of $x^3 - 9x + 1 = 0$ (Three iterations only)

e) Solve by Newton-Raphson method $x^3 + 2x - 20 = 0$ (Three iterations only)

f) Find approximate value of $\sqrt[3]{100}$ by using Newton-Raphson method (Three iterations only)

**6. Attempt any FOUR of the following:** **16**

a) Differentiate $\cos^{-1}(2x\sqrt{1-x2})$ w.r.to $\sec^{-1}\left(\frac{1}{\sqrt{1-x2}}\right)$.

b) If $y = A\cos(\log x) + B\sin(\log x)$, prove that

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

c) Solve by Gauss-elimination method

$x + 2y + 3z = 14,\ 3x + y + 2z = 11,\ 2x + 3y + z = 11$

d) Solve by Jacobi's method

$10x + y + 2z = 13,\ 3x + 10y + z = 14,\ 2x + 3y + 10z = 15$

(Three iterations only)

e) Solve by using Gauss-Seidal method

$6x + y + z = 105,\ 4x + 8y + 3z = 155,\ 5x + 4y - 10z = 65$

(Two iterations only)

f) Solve by Gauss-Seidal method

$x + 7y - 3z = -22,\ 5x - 2y + 3z = 18,\ 2x - y + 6z = 22$

(Two iterations only)